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THE MECHANISM OF NONPERIODIC MOTION IN POROUS MEDIA AS A RESULT OF THE ACCUMULATING EFFECT OF NONLINEAR WAVES

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We carried out a theoretical investigation of the nonlinear mechanism of nonperiodic motion in a saturated porous medium under the prolonged action of elastic waves. Within the framework of a generalization of the Frenkel–Biot–Nikolaevskii model, we suggest a technique for constructing a slow (background) solution that takes into account the accumulating effect of fast nonlinear oscillations.

Numerous experimental data (for example, [1-3]), show that prolonged exposure of rocks saturated with fluid to seismoacoustic vibrations leads to the accumulation of nonperiodic changes in them (acoustic flow of the fluid, temperature rise, accumulation of deformations, etc.). A theoretical description of this nonlinear effect and quantitative estimates can be made within the framework of a generalization [4, 5] of the classical Frenkel–Biot–Nikolaevskii model [6] of the propagation of waves in saturated porous media. The use of the method of multiscale expansions [7, 8] allows one, in the process of asymptotic investigation, to isolate different types of motions: slow (background) averaged macromotion and fast (oscillating) micromotion. Naturally, there is interaction between these two types: the propagation of fast (e.g., elastic) waves depends in many respects on the parameters of the background motion and, on the other hand, the averaging of nonlinear oscillations leads to the accumulation of changes in the parameters of the background motion. In the present work we propose a method for constructing a background solution allowing one to evaluate the accumulating effect of nonlinear elastic (longitudinal of the 1st and 2nd kinds and transverse) waves.

We will consider a viscoelastic deformable porous medium consisting of an elastic skeleton, a viscous fluid bound by the skeleton surface, and a viscous fluid (weakly compressible liquid or perfect gas) phase.

Note that in contrast to the traditional investigation of a porous fluid for porous media, when $P_{ij} = -p\delta_{ij}$, here we take shear stresses in the fluid into account:

$$P_{ij} = -p\delta_{ij} + \nu_f\,[\partial v_{fi}/\partial x_j + \partial v_{fj}/\partial x_i - (2/3)\,(\partial v_{fk}/\partial x_k)\,\delta_{ij}]\,.$$

In a traditional linear consideration the corresponding small terms of equations are usually neglected; therefore, the problem with allowance for the tensor of viscous stresses in a nonlinear statement was not considered earlier. However, for analysis of the dynamics of porous media over long time periods, it is necessary to take into account the accumulated contribution of weak nonlinear and dispersion effects.

The porous medium skeleton and the bound fluid form an effective viscoelastic solid phase that exhibits the elastic properties of the skeleton and the viscous properties of the fluid. In this case the skeleton and bound fluid have the same velocities, temperatures, and pressures.

We introduce into consideration the tensor of effective stresses:

$$\sigma_{ij} = (1 - (1 - \alpha)\,m)\,(\sigma^s_{ij} - P_{ij})\,,$$

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which is interpreted as the difference between the true phase stresses in the solid and fluid phases.

With allowance for viscoelastic properties, the rheological relation for the solid phase can be represented as

$$\sigma_{ij} = Ke_{kk}\delta_{ij} + 2G(e_{ij} - e_{kk}\delta_{ij}/3) + \beta_s Kp\delta_{ij} - \varphi_s KT_s\delta_{ij} +$$

$$+ \alpha m\nu_\alpha [\partial v_{si}/\partial x_j + \partial v_{sj}/\partial x_i - (2/3)(\partial v_{sk}/\partial x_k)\delta_{ij}].$$

Next, we formulate a system of determining equations in dimensionless form. We introduce the following dimensionless variables and parameters:

$$x' = x/x_0,\ t' = t/t_0,\ u' = u/x_0,\ \rho' = \rho/\rho_0,\ P' = P/K_0,\ \sigma' = \sigma/K_0,$$

$$T' = T/\theta_0,\ \beta' = \beta K_0,\ \varphi' = \varphi\theta_0,\ K' = K/K_0,\ G' = G/K_0,\ v' = v/v_0,$$

$$E' = E/v_0^2,\ \nu' = \nu/(K_0 t_0),\ \lambda' = \lambda\theta_0 t_0/(x_0^2 v_0^2 \rho_0),\ C' = C\theta_0/v_0^2,$$

$$R' = R\theta_0/v_0^2,\ \chi' = \chi\theta_0 t_0/(v_0^2\rho_0),\ k' = kx_0,\ \omega' = \omega t_0,\ \text{where}\ x_0 = v_0 t_0,$$

$$t_0 = \rho_0\kappa/\nu_f,\ v_0 = (K_0/\rho_0)^{1/2}.$$

Estimates of the values of the dimensionless parameters can be obtained using characteristic values of the constants of rocks [6]: $K_0 \sim 10^8-10^9$ Pa; $\theta_0 \sim 10^2-10^3$ K; $\rho_0 \sim 10^3$ kg/m^3; $\beta \sim 10^{-10}-10^{-9}$ Pa^{-1}; $\varphi \sim 10^{-6}-10^{-3}$ K^{-1}; $C \sim 10^3$ J/(kg·K); $\lambda \sim 10^0$ W/(m·K), $\nu \sim 10^{-5}-10^{-3}$ Pa·sec; $\kappa \sim 10^{-15}-10^{-12}$ m^2. At these parameter values we have: $t_0 \sim 10^{-9}-10^{-6}$ sec, $v_0 \sim 10^3$ m/sec, $x_0 \sim 10^{-6}-10^{-3}$ m, $\nu' \sim 10^{-8}-10^{-5}$, $\beta' \sim 10^{-2}-10^0$, $\varphi' \sim 10^{-3}-10^0$, $C' \sim 10^0$, $\lambda' \sim 10^{-7}-10^{-4}$.

The dimensional analysis carried out made it possible to isolate the small dimensionless parameter

$$\varepsilon = (\nu_f')^{1/2} = (\nu_f/(K_0 t_0))^{1/2} \equiv \nu_f/(K_0\rho_0\kappa)^{1/2},$$

which represents a combination of the characteristic values of the viscosity of the fluid and the elasticity modulus, density, and permeability of the medium.

In what follows the primes will be omitted. Proceeding from the estimates given above for the dimensionless parameters, we further assume that $\nu = \varepsilon^2\tilde{\nu}$ and $\lambda = \varepsilon^2\tilde{\lambda}$. Omitting the tildes, we write the system of mass, momentum, and energy conservation equations as $(i, j = 1, 2, 3)$

$$\partial((1-\alpha)m\rho_f)/\partial t + \nabla_x((1-\alpha)m\rho_f v_f) = 0;$$

$$\partial(\alpha m\rho_\alpha + (1-m)\rho_s)/\partial t + \nabla_x((\alpha m\rho_\alpha + (1-m)\rho_s)v_s) = 0;$$

$$(1-\alpha)m\rho_f[\partial/\partial t + \langle v_f, \nabla_x\rangle]v_{fi} - (1-\alpha)m\partial P_{ij}/\partial x_j + m^2(1-\alpha)^2(v_{fi} - v_{si}) = 0;$$

$$(\alpha m\rho_\alpha + (1-m)\rho_s)[\partial/\partial t + \langle v_s, \nabla_x\rangle]v_{si} - \partial\sigma_{ij}/\partial x_j - (1-(1-\alpha)m)\partial P_{ij}/\partial x_j -$$

$$- m^2(1-\alpha)^2(v_{fi} - v_{si}) = 0;\quad \partial u_i/\partial t - v_{si} = 0; \tag{1}$$

$$\partial\sigma_{ij}/\partial t = K\delta_{ij}\partial e_{kk}/\partial t + 2G\partial(e_{ij} - e_{kk}\delta_{ij}/3)/\partial t + \beta_s\delta_{ij}K\partial p/\partial t -$$

$$- \varphi_s\delta_{ij}K\partial T_s/\partial t + \varepsilon^2\alpha m\nu_\alpha\partial(\partial v_{si}/\partial x_j + \partial v_{sj}/\partial x_i -$$

$$- (2/3)\,(\partial v_{sk}/\partial x_k)\,\delta_{ij})/\partial t\,; \quad \partial e_{ij}/\partial t - (\partial v_{si}/\partial x_j + \partial v_{sj}/\partial x_i)/2 = 0\,;$$

$$m\,(1-\alpha)\,\rho_f\,[\partial/\partial t + \langle v_f, \nabla_x\rangle]\,E_f = m\,(1-\alpha)\,P_{ij}\partial v_{fi}/\partial x_j +$$

$$+ m^2\,(1-\alpha)^2\,|v_f - v_s|^2 - \chi\,(T_f - T_s) + \varepsilon^2\nabla_x\,(m\,(1-\alpha)\,\lambda_f\nabla_x)\,T_f\,;$$

$$(1-m)\,\rho_s\,[\partial/\partial t + \langle v_s, \nabla_x\rangle]\,E_s + am\rho_\alpha\,[\partial/\partial t + \langle v_s, \nabla_x\rangle]\,E_\alpha =$$

$$= (\sigma_{ij} + (1-(1-\alpha)\,m)\,P_{ij})\,\partial v_{si}/\partial x_j + \chi\,(T_f - T_s) +$$

$$+ \varepsilon^2\nabla_x\,((1-m)\,\lambda_s + am\lambda_\alpha)\,\nabla_x T_s\,.$$

The rheological and thermodynamic relations have the form

$$\rho_{\text{liq}} = \rho_{\text{liq}0}\,(1 + \beta_{\text{liq}}\,(p - p_0) - \varphi_{\text{liq}}\,(T_{\text{liq}} - T_{\text{liq}0}))\,; \quad \rho_g = p/(RT_g)\,;$$

$$\rho_\alpha = \rho_{\text{liq}0}\,(1 - \beta_\alpha\,(\overset{s}{\sigma}_{kk}/3 - \sigma_0) - \varphi_\alpha\,(T_s - T_{s0}))\,; \quad v_f = (v_{f1}, v_{f2}, v_{f3})\,;$$

$$\rho_s = \rho_{s0}\,(1 - \beta_s\,(\overset{s}{\sigma}_{kk}/3 - \sigma_0) - \varphi_s\,(T_s - T_{s0}))\,; \quad v_s = (v_{s1}, v_{s2}, v_{s3})\,;$$

$$P_{ij} = -\,p\delta_{ij} + \varepsilon^2 v_f\,[\partial v_{fi}/\partial x_j + \partial v_{fj}/\partial x_i - (2/3)\,(\partial v_{fk}/\partial x_k)\,\delta_{ij}]\,; \tag{2}$$

$$\rho_{\text{liq}} dE_{\text{liq}} = \rho_{\text{liq}} C_{\text{liq}} dT_{\text{liq}} + (p/\rho_{\text{liq}})\,d\rho_{\text{liq}} - \varphi_{\text{liq}} T_{\text{liq}} dp\,; \quad E_g = C_g T_g\,;$$

$$\rho_s dE_s = \rho_s C_s dT_s + \overset{s}{\sigma}_{ij} de_{ij} + \varphi_s T_s d\overset{s}{\sigma}_{kk}/3\,;$$

$$\rho_\alpha dE_\alpha = \rho_\alpha C_\alpha dT_s - \overset{s}{\sigma}_{kk}/(3\rho_\alpha)\,d\rho_\alpha + \varphi_\alpha T_s d\overset{s}{\sigma}_{kk}/3\,,$$

$$\overset{s}{\sigma}_{ij} = \sigma_{ij}/(1-(1-\alpha)\,m) + P_{ij}\,.$$

System (1)-(2) in its main part represents a hyperbolic, closed system of equations in unknown tensor functions σ_{ij} and e_{ij}, vector functions u, v_s, and v_f, and scalar functions m, p, T_s, and T_f.

It should be noted that consideration of the hyperbolic (in its main portion) system of equations and the selection of the form for the asymptotic solution of this system predetermine the maximum number of real roots of the dispersion equation that correspond to the frequencies of the waves in the model considered.

In what follows we shall limit ourselves to consideration of the Cauchy problem with the initial data

$$u_i|_{t=0} = \overset{0}{u}_i\,, \quad v_{si}|_{t=0} = \overset{0}{v}_{si}\,, \quad v_{fi}|_{t=0} = \overset{0}{v}_{fi}\,, \quad m|_{t=0} = \overset{0}{m}\,, \quad p|_{t=0} = \overset{0}{p}\,,$$

$$T_f|_{t=0} = T_f^0\,, \quad T_s|_{t=0} = T_s^0\,, \quad e_{ij}|_{t=0} = [\partial\overset{0}{u}_i/\partial x_j + \partial\overset{0}{u}_j/\partial x_i]/2\,,$$

$$\sigma_{ij}|_{t=0} = Ke_{kk}\delta_{ij}|_{t=0} + 2G\,(e_{ij} - (1/3)\,e_{kk}\delta_{ij})|_{t=0} + \beta_s K\overset{0}{p}\delta_{ij} -$$

$$- \varphi_s K T_s^0 \delta_{ij} + \varepsilon^2 a\overset{0}{m}\,\overset{0}{v}_\alpha\,[\partial\overset{0}{v}_{si}/\partial x_j + \partial\overset{0}{v}_{sj}/\partial x_i - (2/3)\,(\partial\overset{0}{v}_{sk}/\partial x_k)\,\delta_{ij}]\,. \tag{3}$$

The asymptotic solution of problem (1)-(3) is constructed in the form

$$U\,(\tau, x, t) = U_{\text{back}}\,(x, t) + W\,(\tau, x, t)\,,$$

$$U_{back}(x, t) = U^{(0)}_{back}(x, t) + \varepsilon U^{(1)}_{back}(x, t) + \varepsilon^2 U^{(2)}_{back}(x, t) + \dots, \quad (4)$$

$$W(\tau, x, t) = \varepsilon W^{(1)}(\tau, x, t) + \varepsilon^2 W^{(2)}(\tau, x, t) + \dots$$

Here x, t are the slow variables, $\tau = S(x, t)/\varepsilon$ is the fast variable, $S(x, t)$ is the phase, U is the vector-function of the unknown quantities:

$$U(\tau, x, t) = (m, v_{fi}, v_{si}, p, \sigma_{ij}, e_{ij}, T_f, T_s, u_i), \quad i, j = 1, 2, 3.$$

The functions $W^{(j)}(\tau, x, t)$ are 2π-periodic with a zero mean value in τ, so that the function $U_{back}(x, t)$ is the slow background, which is the mean of the solution U of the initial system of equations. Here $U^{(j)}_{back}(x, t)$ and $W^{(j)}(\tau, x, t)$ are the C^∞-functions in corresponding variables.

Below, we consider for simplicity the case of identical background temperatures ($T_f^{(0)} = T_s^{(0)}$) and zero velocities of motion of the solid and fluid phases ($v_{fi}^{(0)} = v_{si}^{(0)} = 0$, $i = 1, 2, 3$).

To construct the background solution $U_{back}(x, t)$ in mod $O(\varepsilon^2)$ it is necessary to determine the function $W^{(1)}(\tau, x, t)$. We substitute Eq. (4) into Eqs. (1)-(2). Having designated by A the symbol of the linearized operator $A(t, x, \partial/\partial t, \nabla x)$ of the initial system of equations against the background $U^{(0)}_{back}$, we obtain

$$(1/\varepsilon) A(S_t, S_x, x, t) W^{(1)}_\tau(\tau, x, t) + O(1) = 0.$$

Hence, it follows that according to mod $O(1)$

$$A(S_t, S_x, x, t) W^{(1)}(\tau, x, t) = 0.$$

Thus, the existence of a nonzero vector-function $W^{(1)}(\tau, x, t)$ is possible only when

$$\text{Det}\, A(S_t, S_x, x, t) = 0. \quad (5)$$

Consequently, we can present the function $W^{(1)}(\tau, x, t)$ in the form of a product of a certain function $U^{(1)}(\tau, x, t)$ and of the null vector $H(x, t)$ of the matrix A:

$$W^{(1)}(\tau, x, t) = U^{(1)}(\tau, x, t) H(x, t), \quad (6)$$

where $A(S_t, S_x, x, t)H(x, t) = 0$.

The determinant of the matrix $A(t, x, -\omega, k)$ is

$$\text{Det}\, A(t, x, -\omega, k) = (1-\alpha)^5 m^4 \mathscr{P}_4 \mathscr{P}_s^2 \rho_{liq}^2 \omega^{17} / (3(1 - (1-\alpha) m))$$

in the case of a porous medium saturated with a liquid and

$$\text{Det}\, A(t, x, -\omega, k) = (1-\alpha)^5 m^4 \mathscr{P}_4 \mathscr{P}_s^2 \rho_g^2 \omega^{17} / (3RT_g^2 (1 - (1-\alpha) m))$$

in the case of a porous medium saturated with a gas. Here, $-\omega$ and k denote the derivatives of S_t and S_x, respectively.

Solving Eq. (5), we find the frequencies $\omega_i = \omega_i(x, t, k)$ of the waves typical of the considered mathematical model.

The even roots of the polynomial $\mathscr{P}_s$ determine the frequencies of transverse waves of different polarization. The fourth-degree polynomial $\mathscr{P}_4$ determines the frequencies of longitudinal (direct and back) waves of the 1st and 2nd kind.

Finding the function $U^{(1)}(\tau, x, t)$ is reduced to constructing a solution by the method of two scales and to solving the nonlinear evolutional Kortevega–De Vries–Burgers equation [9, 10].

The construction of the background solution is carried out by the substitution of Eq. (4) into Eqs. (1)-(2). The limiting linearized system of equations for a zero approximation of the background solution has a simple form:

$$\partial (m^{(0)}\rho_f^{(0)})/\partial t = 0 , \quad \partial (\alpha m^{(0)}\rho_\alpha^{(0)} + (1 - m^{(0)})\, \rho_s^{(0)})/\partial t = 0 ,$$

$$m^{(0)}\partial p^{(0)}/\partial x_i = 0 , \quad \partial\sigma_{ij}^{(0)}/\partial x_j = 0 , \quad \partial e_{ij}^{(0)}/\partial t = 0 , \quad \partial u_i^{(0)}/\partial t = 0 ,$$

$$\partial\sigma_{ij}^{(0)}/\partial t - \delta_{ij}\beta_s K\partial p^{(0)}/\partial t + \delta_{ij}\varphi_s K\partial T_s^{(0)}/\partial t = 0 ,$$

$$m^{(0)}(1 - \alpha)(\rho_{\rm liq}^{(0)}C_{\rm liq}\partial T_{\rm liq}^{(0)}/\partial t + (p^{(0)}/\rho_{\rm liq}^{(0)})\,\partial\rho_{\rm liq}^{(0)}/\partial t - \varphi_{\rm liq}T_{\rm liq}^{(0)}\partial p^{(0)}/\partial t) +$$

$$+ \chi\,(T_{\rm liq}^{(0)} - T_s^{(0)}) = 0$$

or

$$m^{(0)}(1 - \alpha)\,\rho_g^{(0)}C_g\partial T_g^{(0)}/\partial t + \chi\,(T_g^{(0)} - T_s^{(0)}) = 0 ,$$

$$(1 - m^{(0)})\,(\rho_s^{(0)}C_s\partial T_s^{(0)}/\partial t + \varphi_s T_s^{(0)}\partial\sigma^{(0)}/\partial t) + \alpha m^{(0)}(\rho_\alpha^{(0)}C_\alpha\partial T_s^{(0)}/\partial t -$$

$$- (\sigma^{(0)}/\rho_\alpha^{(0)})\,\partial\rho_\alpha^{(0)}/\partial t + \varphi_\alpha T_s^{(0)}\partial\sigma^{(0)}/\partial t) - \chi\,(T_f^{(0)} - T_s^{(0)}) = 0 ,$$

where

$$\rho_{\rm liq}^{(0)} = \rho_{\rm liq0}(1 + \beta_{\rm liq}(p^{(0)} - p_0) - \varphi_{\rm liq}(T_{\rm liq}^{(0)} - T_{\rm liq0})) , \quad \rho_g^{(0)} = p^{(0)}/(RT_g^{(0)}) ,$$

$$\rho_\alpha^{(0)} = \rho_{\rm liq\,0}(1 - \beta_\alpha(\sigma^{(0)} - \sigma_0) - \varphi_\alpha(T_s^{(0)} - T_{s0})) , \quad \rho_s^{(0)} = \rho_{s0}(1 -$$

$$- \beta_s(\sigma^{(0)} - \sigma_0) - \varphi_s(T_s^{(0)} - T_{s0})) , \quad \sigma^{(0)} = \sigma_{kk}^{(0)}/(3\,(1 - (1 - \alpha)\,m^{(0)})) - p^{(0)} .$$

One can easily see that any constant $U_{\rm back}^{(0)} = \text{const}$ can be taken as a zero approximation of the background solution.

For a first approximation of the background solution (mod $O(\varepsilon)$), we obtain a homogeneous hyperbolic system of equations:

$$\partial (m^{(1)}\rho_f^{(0)} + m^{(0)}\rho_f^{(1)})/\partial t + \nabla_x(m^{(0)}\rho_f^{(0)}v_f^{(1)}) = 0 ,$$

$$\partial (\alpha\,(m^{(0)}\rho_\alpha^{(1)} + m^{(1)}\rho_\alpha^{(0)}) + (1 - m^{(0)})\,\rho_s^{(1)} - m^{(1)}\rho_s^{(0)})/\partial t +$$

$$+ \nabla_x((\alpha m^{(0)}\rho_\alpha^{(0)} + (1 - m^{(0)})\,\rho_s^{(0)})\, v_s^{(1)}) = 0 ,$$

$$\rho_f^{(0)}\partial v_{fi}^{(1)}/\partial t + \partial p^{(1)}/\partial x_i + m^{(0)}(1 - \alpha)\,(v_{fi}^{(1)} - v_{si}^{(1)}) = 0 ,$$

$$(\alpha m^{(0)}\rho_\alpha^{(0)} + (1 - m^{(0)})\,\rho_s^{(0)})\,\partial v_{si}^{(1)}/\partial t + (1 - (1 - \alpha)\,m^{(0)})\,\partial p^{(1)}/\partial x_i -$$

$$- \partial\sigma_{ij}^{(1)}/\partial x_j - (m^{(0)})^2(1 - \alpha)^2(v_{fi}^{(1)} - v_{si}^{(1)}) = 0 ,$$

$$\partial e_{ij}^{(1)}/\partial t - [\partial v_{si}^{(1)}/\partial x_j + \partial v_{sj}^{(1)}/\partial x_i]/2 = 0 , \quad \partial u_i^{(1)}/\partial t - v_{si}^{(1)} = 0 ,$$

$$\partial\sigma_{ij}^{(1)}/\partial t - \delta_{ij}K\partial e_{kk}^{(1)}/\partial t - 2G\partial\,(e_{ij}^{(1)} - e_{kk}^{(1)}\delta_{ij}/3)/\partial t -$$

$$- \delta_{ij}\,\beta_s K\partial p^{(1)}/\partial t + \delta_{ij}\,\varphi_s K\partial T_s^{(1)}/\partial t = 0\,,$$

$$m^{(0)}\,(1-\alpha)\,(\rho_{\text{liq}}^{(0)}C_{\text{liq}}\partial T_{\text{liq}}^{(1)}/\partial t + (p^{(0}/\rho_{\text{liq}}^{(0)})\,\partial\rho_{\text{liq}}^{(1)}/\partial t - \varphi_{\text{liq}}T_{\text{liq}}^{(0)}\partial p^{(1)}/\partial t) +$$

$$+ m^{(0)}\,(1-\alpha)\,p^{(0)}\partial v_{\text{liq}i}^{(1)}/\partial x_i + \chi\,(T_{\text{liq}}^{(1)} - T_s^{(1)}) = 0$$

or

$$m^{(0)}\,(1-\alpha)\,\rho_g^{(0)}C_g\partial T_g^{(1)}/\partial t + m^{(0)}\,(1-\alpha)\,p^{(0)}v_{gi}^{(1)}/\partial x_i + \chi\,(T_g^{(1)} - T_s^{(1)}) = 0\,,$$

$$(1 - m^{(0)})\,(\rho_s^{(0)}C_s\partial T_s^{(1)}/\partial t - (\sigma_{ij}^{s})^{(0)}\partial e_{ij}^{(1)}/\partial t + \varphi_s T_s^{(0)}\partial\sigma^{(1)}/\partial t) +$$

$$+ \alpha m^{(0)}\,(\rho_\alpha^{(0)}C_\alpha\partial T_s^{(1)}/\partial t - (\sigma^{(0)}/\rho_\alpha^{(0)})\,\partial\rho_\alpha^{(1)}/\partial t + \varphi_\alpha T_s^{(0)}\partial\sigma^{(1)}/\partial t) -$$

$$- (\sigma_{ij}^{(0)} - (1 - (1-\alpha)\,m^{(0)})\,p^{(0)}\delta_{ij})\,\partial v_{si}^{(1)}/\partial x_j - \chi\,(T_f^{(1)} - T_s^{(1)}) = 0\,.$$

In the problem considered, the Cauchy data (2) are mod $O(\varepsilon)$ equal to zero. Consequently, this hyperbolic system has a unique zero solution $U_{\text{back}}^{(1)} = 0$.

Here, we essentially used the requirement that the functions $W^{(j)}$ are the functions with a zero mean value with respect to τ. Therefore, performing integration of the equations of the first approximation over the period, we obtained a system of equations for $U_{\text{back}}^{(1)}$ which is independent of $W^{(1)}$. Thus, perturbation of the equilibrium state did not lead to a change in the background in the first approximation.

Further, for $U_{\text{back}}^{(2)}(x, t)$ (mod $O(\varepsilon^2)$) we obtain the following system of equations:

$$\partial\,(m^{(2)}\rho_f^{(0)} + m^{(0)}\rho_f^{(2)})/\partial t + \nabla_x\,(m^{(0)}\rho_f^{(0)}v_f^{(2)}) = 0\,;$$

$$\partial\,(\alpha\,(m^{(0)}\rho_\alpha^{(2)} + m^{(2)}\rho_\alpha^{(0)}) + (1 - m^{(0)})\,\rho_s^{(2)} - m^{(2)}\rho_s^{(0)})/\partial t +$$

$$+ \nabla_x\,((\alpha m^{(0)}\rho_\alpha^{(0)} + (1 - m^{(0)})\,\rho_s^{(0)})\,v_s^{(2)}) = 0\,;$$

$$\rho_f^{(0)}\partial v_{fi}^{(2)}/\partial t + \partial p^{(2)}/\partial x_i + m^{(0)}\,(1-\alpha)\,(v_{fi}^{(2)} - v_{si}^{(2)}) +$$

$$+ (1-\alpha)/2\pi \int_0^{2\pi} W_m^{(1)}\,(W_{v_{f,i}}^{(1)} - W_{v_{s,i}}^{(1)})\,d\tau = 0\,;$$

$$(\alpha m^{(0)}\rho_\alpha^{(0)} + (1 - m^{(0)})\,\rho_s^{(0)})\,\partial v_{si}^{(2)}/\partial t - \partial\sigma_{ij}^{(2)}/\partial x_j +$$

$$+ (1 - (1-\alpha)\,m^{(0)})\,\partial p^{(2)}/\partial x_i - (m^{(0)})^2\,(1-\alpha)^2\,(v_{fi}^{(2)} - v_{si}^{(2)}) -$$

$$- m^{(0)}\,(1-\alpha)/2\pi \int_0^{2\pi} W_m^{(1)}\,(W_{v_{f,i}}^{(1)} - W_{v_{s,i}}^{(1)})\,d\tau = 0\,;$$

$$\partial e_{ij}^{(2)}/\partial t - [\partial v_{si}^{(2)}/\partial x_j + \partial v_{sj}^{(2)}/\partial x_i]/2 = 0\,;\;\; \partial u_i^{(2)}/\partial t - v_{si}^{(2)} = 0\,;$$

$$\partial\sigma_{ij}^{(2)}/\partial t - \delta_{ij}K\partial e_{kk}^{(2)}/\partial t - 2G\partial\,(e_{ij}^{(2)} - e_{kk}^{(2)}\delta_{ij}/3)/\partial t -$$

$$- \delta_{ij} \beta_s K \partial p^{(2)} / \partial t + \delta_{ij} \varphi_s K \partial T_s^{(2)} / \partial t = 0 ;$$

$$m^{(0)} (1 - \alpha) (\rho_{\text{liq}}^{(0)} C_{\text{liq}} \partial T_{\text{liq}}^{(2)} / \partial t + (p^{(0)} / \rho_{\text{liq}}^{(0)}) \partial \rho_{\text{liq}}^{(2)} / \partial t -$$

$$- \varphi_{\text{liq}} T_{\text{liq}}^{(0)} \partial p^{(2)} / \partial t) + m^{(0)} (1 - \alpha) p^{(0)} \partial v_{\text{liq}i}^{(2)} / \partial x_i + \chi (T_{\text{liq}}^{(2)} - T_s^{(2)}) +$$

$$+ (m^{(0)})^2 (1 - \alpha)^2 / 2\pi \int_0^{2\pi} | W_{v_{\text{liq}}}^{(1)} - W_{v_s}^{(1)} |^2 d\tau = 0$$

or

$$m^{(0)} (1 - \alpha) \rho_g^{(0)} C_g \partial T_g^{(2)} / \partial t + m^{(0)} (1 - \alpha) p^{(0)} \partial v_{gi}^{(2)} / \partial x_i + \chi (T_g^{(2)} - T_s^{(2)}) +$$

$$+ (m^{(0)} (1 - \alpha))^2 / 2\pi \int_0^{2\pi} | W_{v_g}^{(1)} - W_{v_s}^{(1)} |^2 d\tau = 0 ;$$

$$(1 - m^{(0)}) (\rho_s^{(0)} C_s \partial T_s^{(2)} / \partial t - (\sigma_{ij}^{s})^{(0)} \partial e_{ij}^{(2)} / \partial t + \varphi_s T_s^{(0)} \partial \sigma^{(2)} / \partial t) +$$

$$+ \alpha m^{(0)} [\rho_\alpha^{(0)} C_\alpha \partial T_s^{(2)} / \partial t - (\sigma^{(0)} / \rho_\alpha^{(0)}) \partial \rho_\alpha^{(2)} / \partial t + \varphi_{\text{liq}} T_s^{(0)} \partial \sigma^{(2)} / \partial t) -$$

$$- (\sigma_{ij}^{(0)} - (1 - (1 - \alpha) m^{(0)}) p^{(0)} \delta_{ij}) \partial v_{si}^{(2)} / \partial x_j - \chi (T_f^{(2)} - T_s^{(2)}) = 0 . \quad (7)$$

It is not difficult to see the presence of "sources" in this system, i.e., integral terms, that are mean values of nonlinear terms. For a linear problem the condition of zero means could have led to a homogeneous system of equations that would have only a trivial solution. In the nonlinear case considered, even for zero Cauchy data for the function $U_{\text{back}}^{(2)}(x, t)$, which result from initial data (3), system (7) has a nontrivial solution that explains the slow motion of the medium.

We note that all the results obtained are easily extended to the case of a multiphase solution, which makes it possible to evaluate the joint accumulating effect of simultaneously propagating longitudinal and transverse waves. In this case, integration should be carried out over the entire set of fast variables τ_i, which correspond to wave frequencies ω_i.

As an illustration, we will consider an approximate solution of the traveling-wave type (stationary) of inhomogeneous system of equations (7).

If in Eq. (6) for $W^{(1)}$ we assume $U^{(1)}$ to be equal to the initial oscillation amplitude u_0, then it is not difficult to obtain an estimate of the background solution in order of magnitude without solving the evolutional equation for amplitude $U^{(1)}$. In this case, the integrals in system (7) will be expressed in terms of the coordinates of the null vector H and u_0.

The substitution of $S = kx - \omega t$ from Eq. (7) yields a system of ordinary differential equations with corresponding zero Cauchy data; this system is easily integrated numerically.

Figure 1 presents the results of calculations of a one-dimensional problem for longitudinal waves of the 1st and 2nd kind in a water-saturated porous medium at the following values of dimensionless parameters: $k = (0.001, 0, 0)$, $\alpha = 0$, $m = 0.2$, $\rho_{\text{liq}} = 1$, $\rho_s = 2.5$, $\varphi_{\text{liq}} = 0.2$, $\beta_{\text{liq}} = 0.1$, $\varphi_s = 0.1$, $\beta_s = 0.1$, $C_{\text{liq}} = 1$, $C_s = 1$, $T_{\text{liq}} = 1$, $T_s = 1$, $G = 0.1$, $K = 1$, $p = 0.01$, $\sigma_{ij} = 0.008$ $(i = j)$, $\sigma_{ij} = 0$ $(i \neq j)$, $\chi = 0$, $u_0 = 1$. Here, for the wave of the 1st kind $\omega = 0.00244$, $H_m = 0.01937$, $H_{v_{\text{liq}1}} = 0.36747$, $H_{v_{s1}} = 0.14291$; for the wave of the 2nd kind $\omega = 0.00049$, $H_m = 0.24224$, $H_{v_{\text{liq}1}} = 0.60184$, $H_{v_{s1}} = -0.15521$.

As is seen from Fig. 1, the change in the background values of unknown functions has an explicit nonlinear character at the initial stage and becomes linear with the "onset" of the S phase. It is interesting to note that the net effect of a longitudinal wave of the 1st kind (pressure wave) shows up most strongly in the reduction of pressure

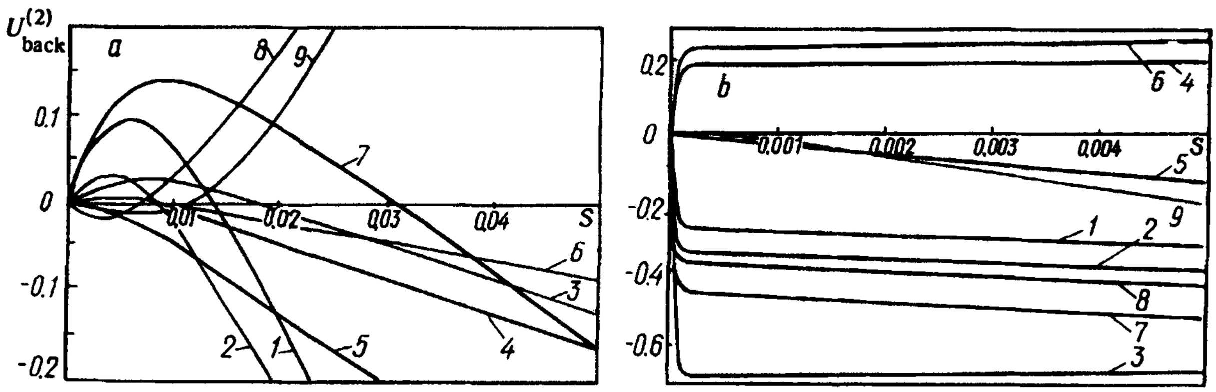

Fig. 1. Distribution of the perturbations $U^{(2)}_{\text{back}}(S)$ of background distributions as a result of the effect of a longitudinal wave of the 1st (a) and 2nd (b) kinds: a) 1, $m \cdot 10^2$; 2) p; 3) v_{liql}; 4) v_{sl}; 5) T_{liq}; 6) T_s; 7) $\sigma_{11} \cdot 10^2$; 8) $e_{11} \cdot 10^1$; 9) u_1. b) 1) m; 2) p; 3) v_{liql}; 4) v_{sl}; 5) $T_{\text{liq}} \cdot 10^{-1}$; 6) $T_s \cdot 10^1$; 7) σ_{11}; 8) e_{11}; 9) $u_1 \cdot 10^{-1}$.

in the liquid and leads to the onset of singly directed motions of the liquid and solid phases. A wave of the 2nd kind (rearrangement wave) leads to more substantial changes in porosity, stresses, deformation, and displacement, as well as to oppositely directed motion of the liquid and solid phases. It is interesting to note that in the model example considered the cooling of the liquid is determined by the predominant process of its expansion.

Thus, allowance for the nonlinearity of oscillations in a saturated porous medium with propagating fast elastic waves makes it possible to explain the slow nonperiodic motion and obtain qualitative estimates of its parameters.

NOTATION

x, space coordinate; t, time; m, porosity; κ, permeability; α, volumetric fraction of bound liquid; ρ, density; P_{ij}, tensor of stresses in fluid phase; σ^s_{ij}, tensor of true stresses in solid phase; σ_{ij}, tensor of effective stresses; v, velocity vector; p, pressure; T, temperature; u, displacement vector; e_{ij}, tensor of deformations; K, modulus of volumetric elasticity; G, shear modulus; β, compressibility factor; φ, coefficient of thermal expansion; E, internal energy; ν, viscosity; λ, thermal conductivity; C, heat capacity; R, universal gas constant; χ, coefficient of interphase heat exchange; k, wave vector; ω, frequency. Subscripts: s, solid phase; f, fluid phase; liq, liquid; g, gas; α, bound liquid.

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